Direct Determination of NMR Correlation Times from Spin−Lattice and Spin−Spin Relaxation Times

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The NMR rotational correlation equations for dipolar relaxation between ^{1}H and the nuclei, ^{1}H, ^{13}C, ^{31}P, and ^{113}Cd have been solved for viscous solutions using the R_2/R_1 dipolar ratio. The rotational correlation times have been determined over the dipolar R_2/R_1 range of 1.1−1200 at field strengths of 4.7, 6.35, 7.05, 9.4, 11.75, and 14.1 T. The calculated correlation times at each field strength have been fitted to pairs of polynomials which reproduce the correlation times from R_2/R_1 values at a given temperature. The ^{13}C−^{1}H and ^{1}H−^{1}H equations are used to determine the ^{27}Al nuclear quadrupole coupling constants in 1:2 LiCl−ethylaluminum dichloride melts and neat ethylaluminum dichloride. The ^{113}Cd−^{1}H and ^{31}P−^{1}H polynomials are used to characterize Cd^{2+} and inorganic phosphate binding sites in the enzyme, yeast inorganic pyrophosphatase.

Introduction

NMR relaxation studies can provide useful information about the dynamics and structure of chemical systems. This information includes such properties as rotational correlation times, internuclear distances, and quadrupolar coupling constants. Of particular interest are the determination of rotational correlation times in the liquid state which can be used to determine a reasonably accurate value of the nuclear quadrupole coupling constant, $\chi = e^2qQ/h$.

Until recently, the determination of rotational correlation times outside of the region of extreme narrowing ($\omega\tau < 1$) has been confined to low-temperature studies ($\omega\tau > 1$) where the correlation equation passes through a minimum and can be solved directly.[1,2] The simplification of NMR correlation equations for quadrupolar nuclei in viscous media has allowed investigators to determine liquid-state correlation times and consequently nuclear quadrupole coupling constants for these nuclei in molecules and molecular complexes.[3−6] The quadrupolar rotational correlation times are then used with viscosity and diffusion measurement to probe the microdynamics of highly viscous molten salts systems.[7,8] The success of this approach lies in the fact that the ratio of the relaxation rates for a quadrupolar nucleus, $R_1(=1/T_1)/R_2(=1/T_2)$, can be represented by a quartic equation that reduces to a simple quadratic.[3−6]

A similar approach is also useful for spin $^1/_2$ nuclei such as ^{13}C, ^{1}H, ^{31}P, and ^{113}Cd that undergo dipolar relaxation with neighboring protons or other heteronuclear nuclei. In these cases however, both the relaxation mechanisms and the correlation equations for $R_1(=1/T_1)$ and $R_2(=1/T_2)$ are somewhat more complicated.[9,10] In this study, these relaxation equations are solved, yielding rotational correlation times for ^{13}C−^{1}H, ^{31}P−^{1}H, ^{113}Cd−^{1}H, and ^{1}H−^{1}H dipolar relaxation equations at magnetic field strengths of 4.70, 6.35, 7.05, 9.4, 11.75, and 14.1 T. The values for the dipolar correlation times are then fitted to a series of polynomial equations that can be used by an investigator, once the ratio of $(R_2/R_1 = T_1/T_2)_{\text{dipolar}}$ is known. The range of values for the $(R_2/R_1)_{\text{dipolar}}$ ratio varies from 1.1 to 1200 using two sets of polynomials that accurately calculate the rotational correlation times (cc = correlation coefficient ≥ 0.999 for all the polynomials included herein).

Experimental Section

Materials. Ethylaluminum dichloride (mp 32 °C) was purified as described previously.[1] All manipulations were performed in a helium atmosphere drybox where samples were loaded into 5-mm sample tubes, capped, and sealed with parafilm. They were then removed from the drybox and sealed immediately with a torch. All glassware was treated with EDTA to remove paramagnetic impurities.

NMR Measurements. ^{1}H and ^{13}C NMR spectra were recorded on a Varian XL-300 spectrometer at 299.94 and 75.43 MHz. Temperature measurements were calibrated against methanol or ethylene glycol and are accurate to within 0.5 °C. Pulse widths were typically 4−10 μs, and longitudinal relaxation times were measured by the inversion−recovery method (180°−τ−90°−T_d) with $T_d > 10T_1$. For all melt samples, at least 12 delay times (τ) were used and relaxation times (in duplicate) obtained from a three-parameter exponential fit of magnetization as a function of τ. We observed single exponential behavior for all measurements. T_2's for were determined from half-height peak widths($=1/\pi T_2^*$, where $T_2^* \approx T_2$).

Theoretical Section

Relaxation Mechanisms. The ^{13}C spin−lattice relaxation rate ($R_1 = 1/T_1$) often provides useful information concerning molecular dynamics in solution.[2,11−18] The relaxation mechanisms that can contribute to the spin−lattice ^{13}C relaxation rates (R_1) include dipole−dipole, chemical shift anisotropy, spin rotation, scalar relaxation, chemical exchange, and paramagnetic relaxation. Of these various relaxation mechanisms, dipolar relaxation can be directly related to rotational motion with the use of spherical harmonic functions.[10,11] Solution of the resulting autocorrelation functions produces spectral density functions, assuming exponential decay.[10,11] The resulting spectral density equations[10] used herein are correlation time dependent at a set frequency.[10,11]

Heteronuclear Dipolar Relaxation. In a manner similar to the relaxation between ^{13}C and ^{1}H, ^{31}P and ^{113}Cd can also be relaxed by neighboring ^{1}H atoms. An essential requirement for relaxation is that interactions causing fluctuations at or near the Larmor frequency will be the most effective in causing relaxation. Consequently, the correlation time equation for dipolar spin−lattice relaxation includes terms that are within

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the range of resonance frequencies. The intramolecular dipole−dipole (rotational motion) contribution to spin−lattice relaxation for unlike nuclei of spin $^1/_2$ such as ^{13}C, ^{31}P, or ^{113}Cd being relaxed by neighboring hydrogens is as follows:[10]

$$R_1 = [N_H\gamma_X^2\gamma_H^2\hbar^2/10r_{XH}^6][\tau_c/(1+\omega_-^2\tau_c^2) + 3\tau_c/(1+\omega_X^2\tau_c^2) + 6\tau_c/(1+\omega_+^2\tau_c^2)] \quad (1)$$

where N_H is the number of hydrogen atoms attached to (or interacting with) X (^{13}C, ^{31}P, or ^{113}Cd), γ_X is the magnetogyric ratio of ^{13}C, ^{31}P, or ^{113}Cd, γ_H is the magnetogyric ratio of ^{1}H, $\omega_H = 2\pi\nu_H$, $\omega_X = 2\pi\nu_X$, $\omega_- = \omega_H - \omega_X$, $\omega_+ = \omega_H + \omega_X$, r_{XH} is the distance between ^{13}C, ^{31}P, or ^{113}Cd and a neighboring ^{1}H, and τ_c is the effective correlation time which usually varies exponentially with temperature

Both spin−lattice and spin−spin relaxation are affected by rotational and diffusional motion in liquids. As in the case of dipolar spin−lattice relaxation, dipolar spin−spin relaxation can be directly related to rotational motion with the use of spherical harmonic functions. In addition to the rotational and diffusional motions that affect spin−lattice relaxation, random forces which modulate the spin energy levels at very low frequencies contribute to spin−spin lattice relaxation without inducing transitions. Consequently, the dipolar spin−spin relaxation correlation time equation contains a zero-frequency spectral density term that is not present in eq 1. The intramolecular dipole−dipole (rotational motion) contribution to spin−spin relaxation for unlike nuclei of spin $^1/_2$ such as ^{13}C, ^{31}P, or ^{113}Cd being relaxed by neighboring hydrogens is[10]

$$R_2 = [N_H\gamma_X^2\gamma_H^2\hbar^2/20r_{XH}^6][4\tau_c + \tau_c/(1+\omega_-^2\tau_c^2) + 3\tau_c/(1+\omega_X^2\tau_c^2) + 6\tau_c/(1+\omega_+^2\tau_c^2) + 6\tau_c/(1+\omega_H^2\tau_c^2)] \quad (2)$$

where N_H is the number of hydrogen atoms attached to (or interacting with) X (^{13}C, ^{31}P, or ^{113}Cd), γ_X is the magnetogyric ratio of ^{13}C, ^{31}P, or ^{113}Cd, γ_H is the magnetogyric ratio of ^{1}H, $\omega_H = 2\pi\nu_H$, $\omega_X = 2\pi\nu_X$, $\omega_- = \omega_H - \omega_X$, $\omega_+ = \omega_H + \omega_X$, r_{XH} is the distance between ^{13}C, ^{31}P, or ^{113}Cd and a neighboring ^{1}H, and τ_c is the effective correlation time which usually varies exponentially with temperature

Homonuclear Dipolar Relaxation. The most important relaxation mechanism between protons in liquids is dipolar relaxation.[19−22] Derivation of the intramolecular dipolar spin−lattice relaxation correlation time equation for hydrogens being relaxed by other hydrogens results in the following:[9,10]

$$R_1 = [3\gamma_H^4\hbar^2/10r_{HH}^6][\tau_c/(1+\omega_H^2\tau_c^2) + 4\tau_c/(1+\omega_H^2\tau_c^2)] \quad (3)$$

where γ_H is the magnetogyric ratio of ^{1}H, $\omega_H = 2\pi\nu_H$, r_{HH} is the distance between neighboring ^{1}H's, and τ_c is the effective correlation time.

The intramolecular dipolar spin−spin relaxation correlation time equation for homonuclear spin systems includes a zero-frequency term to account for random forces which modulate the spin energy levels at very low frequencies without inducing transitions.[10] The resulting equation for ^{1}H−^{1}H intramolecular dipolar spin−spin relaxation as a function of correlation time is[9,10]

$$R_2 = [3\gamma_H^4\hbar^2/20r_{HH}^6][3\tau_c + 5\tau_c/(1+\omega_H^2\tau_c^2) + 2\tau_c/(1+4\omega_H^2\tau_c^2)] \quad (4)$$

Dipolar Relaxation Ratios. The solution of eqs 1−4 is often difficult unless the appropriate correlation equation can be solved at a T_1 or T_2 minimum, suggesting low-temperature studies. Often this is either inconvenient or impossible for the investigator, thus eliminating relaxation studies as a useful source of information. In this report, we offer an alternative method that provides rotational correlation times in viscous media, subject to certain requirements.

In cases where dipolar relaxation is the major contributing relaxation mechanism as is often the case with ^{1}H−^{1}H [19−22] and ^{13}C−^{1}H (where the carbon is directly bonded to a neighboring hydrogen) relaxation or if the fraction of dipolar contribution to the spin−lattice and spin−spin relaxation mechanisms is essentially the same for each mechanism, then a ratio of spin−spin to spin−lattice relaxation times approach can be used to determine rotational correlation times. As in the case of quadrupolar nuclei,[3−6] the ratio of R_1/R_2 (eq 1/eq 2 or eq 3/eq 4) does eliminate a number of terms contained in eqs 1−4. The resulting eqs 5 (=eq 1/eq 2) and 6 (=eq 3/eq 4) can be solved by iterative methods, assuming a range of values for R_1/R_2. Once the iterative solutions of eqs 5 and 6 are known at each field strength, it is then possible to represent these solutions by a separate polynomial for each field strength. Knowledge of this polynomial then allows the investigator to determine rotational correlation times at a particular field strength from a limited number of measurements:

$$R_1/R_2 = T_2/T_1 = [2/(1+\omega_-^2\tau_c^2) + 6/(1+\omega_X^2\tau_c^2) + 12/(1+\omega_+^2\tau_c^2)]/[4 + 1/(1+\omega_-^2\tau_c^2) + 3/(1+\omega_X^2\tau_c^2) + 6/(1+\omega_+^2\tau_c^2) + 6/(1+\omega_H^2\tau_c^2)] \quad (5)$$

$$R_1/R_2 = T_2/T_1 = [2/(1+\omega_H^2\tau_c^2) + 8/(1+4\omega_H^2\tau_c^2)]/[3 + 5/(1+\omega_H^2\tau_c^2) + 2/(1+4\omega_H^2\tau_c^2)] \quad (6)$$

Results

Solutions of Eqs 5 and 6. The solutions of eqs 5 (^{13}C, ^{31}P, and ^{113}Cd relaxed by ^{1}H) and 6 (^{1}H relaxed by ^{1}H) at field strengths of 4.70, 6.35, 7.05, 9.4, 11.75, and 14.1 T can be accurately (cc = 0.999) represented by two series of polynomial equations for R_2/R_1 values from 1.10 to 20 and 20 to 1000. Typical plots of the polynomials (cc = 0.999) that cover the two ranges of R_2/R_1 are shown in Figures 1 and 2 for ^{13}C relaxed by ^{1}H over the entire range of field strengths (4.70, 6.35, 7.05, 9.4, 11.75, and 14.1 T).

The intercepts and polynomial coefficients that can be used for the calculation of correlation times for R_2/R_1 ratios are given in Tables 1 and 2. Equation 7 is the form of the polynomial used for the calculation of correlation times.

$$\tau_c(\text{ns}) = a_0 + a_1(R_2/R_1) + a_2(R_2/R_1)^2 + a_3(R_2/R_1)^3 + a_4(R_2/R_1)^4 \quad (7)$$

^{13}C NMR Correlation Times. LiCl−ethylaluminum dichloride (1:2) room-temperature melts have provided a model system for the determination of ^{7}Li and ^{27}Al quadrupole coupling constants in the liquid state.[4,6] By determining the R_2/R_1 ratio for ^{27}Al and ^{7}Li in this viscous system and solving a quartic equation that reduces to a simple quadratic, it was possible to determine correlation times over a wide range of temperatures.[4,6] These same correlation times were then used to calculate values of the ^{27}Al and ^{7}Li quadrupole coupling constant at each temperature, assuming that the asymmetry parameter of the electric field gradient tensor is negligible.[4,6] This latter as-

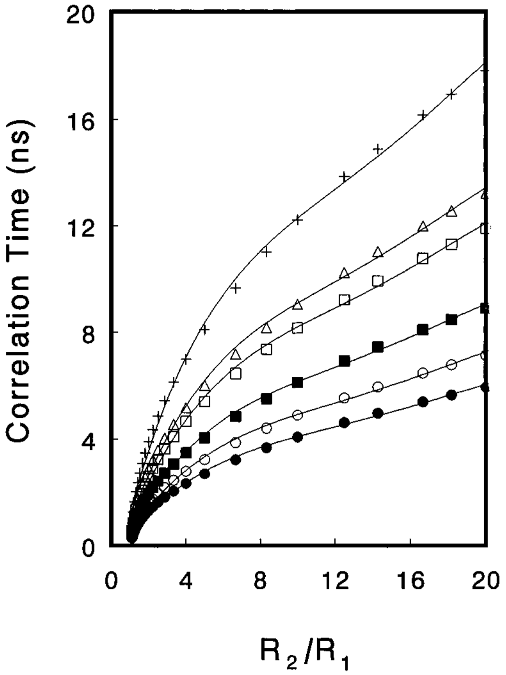

Figure 1. NMR ^{13}C correlation times (ns) at 14.1 (●), 11.75 (○), 9.4 (■), 7.05 (□), 6.35 (△), and 4.70 T (+) vs R_2/R_1 from 0 to 20.

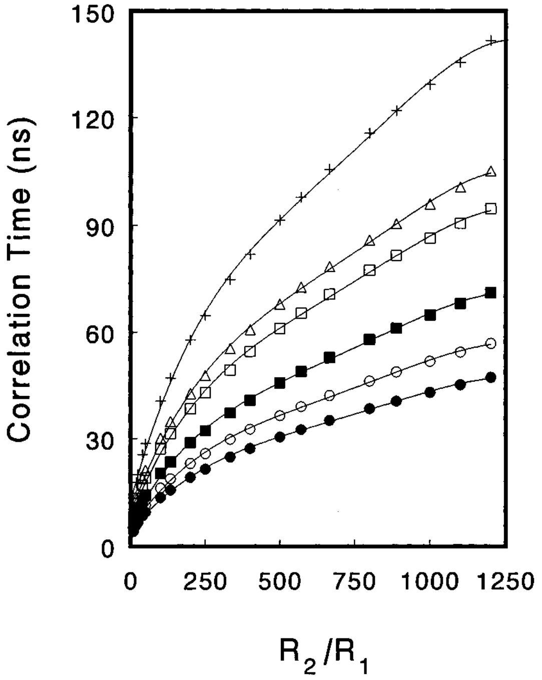

Figure 2. NMR ^{13}C correlation times (ns) at 14.1 (●), 11.75 (○), 9.4 (■), 7.05 (□), 6.35 (△), and 4.70 T (+) vs R_2/R_1 from 20 to 1200.

sumption introduces a maximum error of 15% in the value of the nuclear quadrupole coupling constants.[4]

The quadrupole coupling constant is a sensitive indicator of changes in molecular and/or ionic symmetry.[23] Both the ^{7}Li and ^{27}Al quadrupole coupling constants in 1:2 LiCl−ethylaluminum dichloride room-temperature melts[4,6] decrease in value as the temperature falls below 15 °C, indicating an approach to cubic symmetry about each metallic nucleus.[24] Using Tables 1 and 2, ^{13}C correlation times have been calculated between 75 and −25 °C for the methylene and methyl carbons of the ethyl groups in the 1:2 LiCl−ethylaluminum dichloride room-temperature melt. The results are given in Figure 3 in addition to the ratio of methyl to methylene correlation times. The correlation times reach a minimum between 5 and 25 °C, the region in which one observes a maximum for both the ^{7}Li and ^{27}Al quadrupole coupling constants.[4,6] Between 75 and 60 °C a steady decrease in the ratio of methyl to methylene correlation times is observed, indicating a possible tightening of the melt's structure. At lower temperatures (between 32—the melting point of ethylaluminum dichloride—and 5 °C), a perturbation in the ratio of methyl to methylene correlation times is observed. This perturbation then disappears, and a gradual increase in the ratio is observed as the temperature is lowered further. This final result is consistent with restricted motion of the ethyl group within the 1:2 LiCl−ethylaluminum dichloride melt.

^{1}H Correlation Times. Ethylaluminum dichloride exists in the liquid state as a dimer with D_{2h} symmetry.[25,26] An approximate value of 71.9 MHz for the liquid-state ^{27}Al quadrupole coupling constant (χ) of the ethylaluminum dichloride dimer has been reported previously[27] using the dual spin probe (DSP) method.[28] This approximate ^{27}Al χ value of 71.9 MHz is impossibly large when compared with the ^{27}Al quadrupole coupling constants reported for similar compounds in a low-temperature ^{27}Al NMR study.[1]

In an attempt to better approximate the ^{27}Al χ value for the ethylaluminum dichloride dimer, measurements of T_1 and T_2 for the protons on the methyl carbon over the temperature range of 35 (mp ≈ 32 °C) to 75 °C give R_2/R_1 ratios between 5.5 and 28.8. It is unlikely that the protons on both the methylene and methyl groups lie on the principal axis of the electric field gradient tensor of ^{27}Al. Consequently, our resulting correlation times may be improved over previous values but are still only approximate.

Using eq 7, one obtains correlation times between 2.6 and 3.4 ns as shown in Figure 4. ^{27}Al quadrupolar R_1 values between 14.6 and 6.8 ms^{-1} have been reported over this same range of temperatures for the ethylaluminum dichloride dimer.[27] These ^{27}Al quadrupolar relaxation rates[27] and methyl proton (of the attached ethyl group) correlation times are used with eq 8 (assuming that the asymmetry parameter, $0 \leq \eta \leq 1$, of the electric field gradient tensor is 0) to calculate the ^{27}Al quadrupole coupling constants in Figure 4:

$$R_1^{Q} = 1/T_1^{Q} = [3\pi^2(2I+3)/10I^2(2I-1)][1+(\eta^2/3)][e^2Qq/h]^2\tau_c \quad (8)$$

where $I = 5/2$ for ^{27}Al, eQ is the nuclear electric quadrupole moment, eq is the maximum component of the field gradient tensor, and $\chi = e^2Qq/h$.

The region between 50 and 60 °C is marked by an increase in both the ^{1}H correlation time and the ^{27}Al quadrupole coupling constant. This is the same temperature range where a variation in the solution structure of the ethylaluminum dichloride dimer has been previously observed.[27] The χ values obtained from eq 8 for the ethylaluminum dichloride dimer ^{27}Al range from 6.4 to 4.9 MHz between 75 and 35 °C. These values of the ^{27}Al quadrupole coupling constants are comparable to those values obtained for the 1:2 LiCl−ethylaluminum dichloride complex (3.4−3.1 MHz) over the same temperature range.

^{113}Cd Correlation Times. ^{113}Cd has been used as a metal probe for many metalloproteins because of its excellent spectroscopic properties and its ability to replace Ca^{2+}.[29,30] The usefulness of ^{113}Cd as a metal ion probe in biological systems is due to the ability of Cd^{2+} to form complexes with many different conformations and ligand numbers. In addition, Cd-substituted metalloproteins generally retain at least some biological activity. Furthermore, the ^{113}Cd chemical shift covers a wide range from 750 to −200 ppm and the nature of the

TABLE 1: ^{13}C, ^{31}P, ^{113}Cd, and ^{1}H Coefficients for Eq 7 over the R_2/R_1 Range of 1−20

^{13}C−^{1}H R_2/R_1 Range of 1−20

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	−0.512629	−0.613279	−0.767620	−1.022697	−1.137404	−1.548707
a_1	1.017718	1.220673	1.526649	2.035180	2.261762	3.055627
a_2	−0.091038	−0.109236	−0.136774	−0.182156	−0.202486	−0.273711
a_3	0.004246	0.005097	0.006389	0.008502	0.009452	0.012777
a_4	−0.000071	−0.000085	−0.000107	−0.000142	−0.000158	−0.000214

^{31}P−^{1}H R_2/R_1 Range of 1−20

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	−0.370426	−0.442971	−0.556201	−0.742369	−0.823998	−1.112475
a_1	0.703233	0.842681	1.056112	1.408327	1.564261	2.111038
a_2	−0.063096	−0.075561	−0.094990	−0.126596	−0.140525	−0.189580
a_3	0.002916	0.003491	0.004399	0.005857	0.006499	0.008765
a_4	−0.000048	−0.000058	−0.000073	−0.000097	−0.000108	−0.000145

^{113}Cd−^{1}H R_2/R_1 Range of 1−20

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	−0.567419	−0.684285	−0.847273	−1.140389	−1.265277	−1.707633
a_1	1.123500	1.350842	1.685239	2.251383	2.500130	3.374873
a_2	−0.098814	−0.119070	−0.147489	−0.198461	−0.220280	−0.297279
a_3	0.004559	0.005500	0.006783	0.009166	0.010172	0.013725
a_4	−0.000076	−0.000091	−0.000112	−0.000152	−0.000169	−0.000228

^{1}H−^{1}H R_2/R_1 Range of 1−20

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	−0.120440	−0.144546	−0.180407	−0.240395	−0.266772	−0.360547
a_1	0.197251	0.236701	0.295565	0.394030	0.437454	0.591041
a_2	−0.015300	−0.018336	−0.022874	−0.030491	−0.033800	−0.045739
a_3	0.000666	0.000796	0.000993	0.001324	0.001465	0.001987
a_4	−0.000011	−0.000013	−0.000016	−0.000021	−0.000023	−0.000032

TABLE 2: ^{13}C, ^{31}P, ^{113}Cd, and ^{1}H Coefficients for Eq 7 over the R_2/R_1 Range of 20−1200

^{13}C−^{1}H R_2/R_1 Range of 20−1200

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	4.180273	5.016689	6.267684	8.397854	9.288383	12.527152
a_1	0.102696	0.123165	0.154063	0.204502	0.228218	0.307798
a_2	−0.000161	−0.000193	−0.000242	−0.000320	−0.000358	−0.000484
$a_3 \times 10^7$	1.479606	1.771653	2.219700	2.937471	3.285708	4.431994
$a_4 \times 10^{11}$	−5.000063	−5.984294	−7.500430	−9.937629	−11.100570	−14.974160

^{31}P−^{1}H R_2/R_1 Range of 20−1200

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	2.343334	2.808959	3.517779	4.690726	5.214131	7.042617
a_1	0.075676	0.091069	0.113451	0.151244	0.167973	0.226681
a_2	−0.000013	−0.000157	−0.000194	−0.000259	−0.000288	−0.000388
$a_3 \times 10^7$	1.238987	1.509631	1.856519	2.472987	2.751184	3.703983
$a_4 \times 10^{11}$	−4.292364	−5.253492	−6.431405	−8.558790	−9.538806	−12.829270

^{113}Cd−^{1}H R_2/R_1 Range of 20−1200

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	4.662062	5.595965	7.024128	9.329185	10.360698	13.991018
a_1	0.114625	0.137571	0.172661	0.229256	0.254765	0.343888
a_2	−0.000180	−0.000216	−0.000271	−0.000360	−0.000400	−0.000540
$a_3 \times 10^7$	1.648356	1.980790	2.488168	3.301917	3.669465	4.951154
$a_4 \times 10^{11}$	−5.565787	−6.691932	−8.408122	−11.157340	−12.398680	−16.727810

^{1}H−^{1}H R_2/R_1 Range of 20−1200

	14.1 T	11.75 T	9.4 T	7.05 T	6.35 T	4.70 T
a_0	1.020429	1.226626	1.531249	2.407145	2.271115	3.063019
a_1	0.021899	0.026262	0.032861	0.035445	0.048648	0.065712
a_2	−0.000033	−0.000039	−0.000049	−0.000031	−0.000072	−0.000098
$a_3 \times 10^7$	0.290682	0.347858	0.436269	0.121004	0.645983	0.872292
$a_4 \times 10^{11}$	−0.966117	−1.154571	−1.448567	0	−2.147143	−2.898401

coordinating ligands and their coordination number are strongly reflected in the chemical shift.[29,30]

In an equilibrium dialysis and NMR study of the enzyme, yeast inorganic pyrophosphatase, four binding sites for ^{113}Cd were detected in a solution containing 2.8 mM yeast inorganic pyrophosphatase, 11.2 mM ^{113}Cd^{2+}, and 8.4 mM inorganic phosphate.[31] Of these four binding sites, three are at the active site and the remaining one is a loosely bound ^{113}Cd. The four

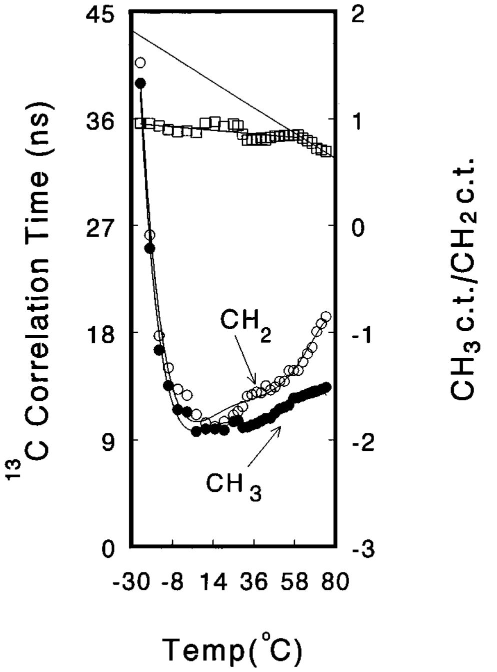

Figure 3. NMR ^{13}C correlation times (ns) for methyl (●) and methylene (○) carbons in 1:2 LiCl−ethylaluminum dichloride and ratio of methyl to methylene carbon correlation times (□) from 75 to −25 °C.

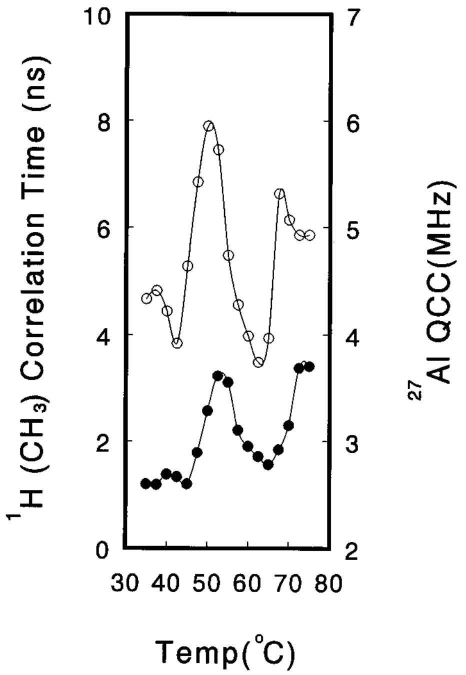

Figure 4. NMR ^{1}H correlation times (ns) for methyl (●) carbons and ^{27}Al quadrupole coupling constants (○) of ethylaluminum dichloride dimer from 35 to 75 °C.

^{113}Cd resonances are located at +12.2, −14.2, −16.5, and −28.4 ppm relative to 0.1 M $Cd(ClO_4)_2$. The R_2/R_1 ratios for these peaks are 432, 750, 600, and 814, respectively. Using the equation for 4.70 T and ^{113}Cd in Table 2, values of 96, 124, 111, and 130 ns are obtained for the ^{113}Cd correlation times. These results suggest that the peak at +12.2 ppm is associated with the loosely bound ^{113}Cd.

^{31}P Correlation Times. In the same study[29] of inorganic pyrophosphatase, three ^{31}P resonances are detected at 9.6, 4.9, and 1.4 ppm relative to 0.1 M H_3PO_4.[31] The R_2/R_1 ratios for these peaks are 986, 1275, and 1001, respectively. Using the equation for 4.70 T and ^{31}P in Table 2, values of 87, 94, and 88 ns are obtained for the ^{31}P correlation times. The similarity in ^{31}P correlation times and the large R_2/R_1 ratios suggest that these three peaks are associated with various P_i species, all of which are bound to the enzyme.

Summary

The NMR dipolar correlation equations for $R_1(=1/T_1)$ and $R_2(=1/T_2)$ are solved jointly for ^{1}H, ^{13}C, ^{31}P, and ^{113}Cd interacting with ^{1}H in viscous solutions. Sets of polynomials have been generated which provide correlation times for these four nuclei undergoing dipolar relaxation with ^{1}H once the ratio of $(R_2/R_1)_{\text{dipolar}}$ is known. These equations are applicable in the region where $\omega\tau > 1$ and $R_2 > R_1$.

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References and Notes

(1) Keller, C. E.; Carper, W. R. *Inorg. Chim. Acta* **1993**, *210*, 203−208.

(2) Keller, C. E., Carper, W. R. *J. Phys. Chem.* **1994**, *98*, 6865−6869.

(3) Decatur, J. D.; Farrar, T. C. *J. Phys. Chem.* **1990**, *94*, 7391−7401.

(4) Keller, C. E.; Piersma, B. J.; Mains, G. J.; Carper, W. R. *Inorg. Chem.* **1994**, *33*, 5601−5603.

(5) Stringfellow, T. C.; Farrar, T. C. *J. Phys. Chem.* **1995**, *99*, 3889−3891.

(6) Keller, C. E., Piersma, B. J.; Carper, W. R. *J. Phys. Chem.* **1995**, *99*, 12998−13001.

(7) Larive, C. K.; Lin, M.; Piersma, B. J.; Carper, W. R. *J. Phys. Chem.* **1995**, *99*, 12409−12412.

(8) Carper, W. R.; Mains, G. J.; Piersma, B. J.; Mansfield, S. L.; Larive, C. K. *J. Phys. Chem.* **1996**, *100*, 4724−4728.

(9) Solomon, I. *Phys. Rev.* **1955**, *99*, 559−565.

(10) Abragam, A. *Principles of Nuclear Magnetism*; Oxford University Press: Oxford, U.K., 1961; Chapter 8.

(11) Farrar, T. C.; Becker, E. D. *Pulse and Fourier Transform NMR. Introduction to Theory and Methods*; Academic Press: New, York, 1971.

(12) Farrar, T. C.; Druck, S. J.; Shoup, R. R.; Becker, E. D. *J. Am. Chem. Soc.* **1972**, *94*, 699−703.

(13) Levy, G. C.; Cargioli, J. D.; Anet, F. A. *J. Am. Chem. Soc.* **1973**, *95*, 1527−1535.

(14) Levy, G. C. *Acc. Chem. Res.* **1973**, *6*, 161−169.

(15) Levy, G. C.; Edlund, U. *J. Am. Chem. Soc.* **1975**, *97*, 5031−5032.

(16) Lyerla, J. R.; Levy, G. C. In *Topics in Carbon-13 NMR Spectroscopy*; Levy, G. C., Ed.; John Wiley: New York, 1976; Vol. 1, p 79.

(17) Wright, D. A.; Axelson, D. E.; Levy, G. C. In *Topics in Carbon-13 NMR Spectroscopy*; Levy, G. C., Ed.; John Wiley: New York, 1979; Vol. 3, p 103.

(18) Craik, D. J.; Levy, G. C. In *Topics in Carbon-13 NMR Spectroscopy*; Levy, G. C., Ed.; John Wiley: New York, 1984; Vol. 4, p 239.

(19) Vold, R. L.; Vold, R. R. In *Progress in NMR Spectroscopy*; Emsley, J. W., Feeney, J., Sutcliffe, L. H., Eds.; Academic Press: New York, 1979; Vol 12, pp 79−133.

(20) Freeman, R.; Wittekoek, S. *J. Magn. Reson.* **1969**, *1*, 238−276.

(21) Freeman, R.; Hill, H. D. W.; Tomlinson, B. L.; Hall, L. D. *J. Chem. Phys.* **1974**, *61*, 4466−4473.

(22) Hall, L. D.; Hill, H. D. W. *J. Am. Chem. Soc.* **1976**, *98*, 1269−1270.

(23) Akitt, J. W. In *Progress in NMR Spectroscopy*; Emsley, J. W., Feeney, J., Sutcliffe, L. H., Eds.; Academic Press: New York, 1988; Vol 21, pp 1−149.

(24) Akitt, J. W.; McDonald, W. S. *J. Magn. Reson.* **1984**, *58*, 401−412.

(25) Weidlein, *J. Organomet. Chem.*, **1969**, *17*, 213−222.

(26) Yamamoto, O.; Hayamizu, K. *J. Organomet. Chem.* **1974**, *73*, 17−25.

(27) Keller, C. E.; Carper, W. R. *J. Magn. Reson., Ser. A* **1994**, *110*, 125−129.

(28) Dechter, J. J.; Henriksson, U.; Kowalewski, J.; Nilsson, A.-C. *J. Magn. Reson.* **1982**, *48*, 503−511.

(29) Summers, M. F. *Coord. Chem. Rev.* **1988**, *86*, 43−134.

(30) Johansson, C.; Drakenberg, T. In *Annual Reports on NMR Spectroscopy*; Webb, G. A., Ed.; Academic Press: New York, 1989; Vol. 22, pp 1−59.

(31) Welsh, K. M.; Cooperman, B. S. *Biochemistry* **1984**, *23*, 4947−4955.